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BOUNDARY LAYER ON A ROTATING CYLINDER IN AXIAL FLOW

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A semiinfinite hollow cylinder of radius R is rotating about its own axis at an angular velocity ω, and an incompressible liquid flows around it in uniform flow at a velocity u_∞. The flow is assumed to be laminar and axisymmetrical. The variables

$$s=\beta\xi;\ \eta=(r^2-R^2)/2\xi R^2;$$
$$\varphi=\psi/u_\infty\xi R^2;\quad w=w^*/\omega R;\quad p=(p^*-p_\infty)/\rho u_\infty^2 \tag{1}$$

are used to solved the problem, where

$$\xi=\sqrt{(\nu x/u_\infty)}/R;\ \beta=(\omega R/u_\infty)^2; \tag{2}$$

ξ and η are analogous to the variables proposed in [2] for the case of a nonrotating cylinder; ψ is the stream function, which is defined by the relations $u^*=\psi_r/r$ and $v^*=-\psi_x/r$; x is the distance from the origin of the cylinder along the generating line; r is the distance to the axis of the cylinder; u^*, v^*, and w^* are the longitudinal, radial, and circumferential components of the velocity; p^* is the pressure; p_∞ is the pressure in the advancing flow; ρ is the density of the liquid; and ν is the kinematic modulus of viscosity. From here on an independent variable which appears as a subscript denotes differentiation with respect to it. The relation $u^*/u_\infty=u=\varphi_\eta$ is valid for the longitudinal velocity component.

In the dynamical equations, terms of the order $(\mathrm{Re}^x)^{-1}=\nu/u_\infty x$ are discarded, i.e., an approximation to the boundary layer is used,

$$2(\sigma\varphi_{\eta\eta})_\eta+\varphi\varphi_{\eta\eta}+\eta p_\eta-sp_s=s(\varphi_\eta\varphi_{\eta s}-\varphi_s\varphi_{\eta\eta});$$
$$2(\sigma w_\eta)_\eta+\varphi w_\eta+(\xi/\sigma)(\varphi-\eta\varphi_\eta+s\varphi_s-2\xi)w=s(\varphi_\eta w_s-\varphi_s w_\eta),$$
$$p_\eta=sw^2/\sigma,\ \sigma=1+2\xi\eta. \tag{3}$$

The flow around the exterior surface is investigated, and the boundary conditions are of the form

$$\varphi_\eta=\varphi=0,\ w=1 \text{ when } \eta=0;$$
$$\varphi_\eta=0,\ w=p=0 \quad\text{as}\quad \eta\to\infty. \tag{4}$$

The case in which the ratio of the thickness of the boundary layer to the radius of the cylinder is small, i.e., $\xi\ll 1$, is of special interest. A limiting transition is possible in Eqs. (3) as $\xi\to 0$ (or as $\beta\to\infty$)

$$2\varphi_{\eta\eta\eta}+\varphi\varphi_{\eta\eta}+\eta p_\eta-sp_s=s(\varphi_\eta\varphi_{\eta s}-\varphi_s\varphi_{\eta\eta});$$
$$2w_{\eta\eta}+\varphi w_\eta=s(\varphi_\eta w_s-\varphi_s w_\eta),\ p_\eta=sw^2. \tag{5}$$

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TABLE 1

i	$u_{\eta 0}^{(i)}$	$w_{\eta 0}^{(i)}$	$p_0^{(i)}$
0	0,33206	—0,33206	0
1	0,68115	—0,14422	—1,0567
2	—0,7982	0,2566	0,4173
3	2,345	—0,8115	—0,8840
4	—9,269	3,284	2,857
5	43,01	—15,36	—11,44
6	—221,7	79,27	52,49
7	—	—	—265,1

It is advisable in the other case of small rotational velocities ($\beta \ll 1$) to use the variables ξ instead of s and p/β instead of p, which allows carrying out the limiting transition as $\beta \to 0$; then the first equation of the system (3) becomes independent and with the boundary conditions for φ from (4) defines the problem of the axial flow around a nonrotating cylinder.

The problem (3) and (4) allows the solution to be represented in the form of function power series arranged in powers of ξ or s. The principal part of the expansion in s is determined by the Blasius function $\Phi(\eta)$

$$\varphi^{(0)} = \Phi; \quad w^{(0)} = 1 - \Phi_\eta; \quad p^{(1)} = \int\limits_\infty^\eta (1 - \Phi_\eta)^2 d\eta,$$

and $\varphi^{(0)}(\eta)$ and $w^{(0)}(\eta)$ represent a solution (derived by Howarth [3]) which does not take account of centrifugal forces and the transverse curvature of the surface of the cylinder. The remaining terms depend on the parameter β, and they are calculated out to the sixth order, inclusively, in the limiting case of a thin boundary layer (5). The values of the quantities which determine the longitudinal and circumferential coefficients of friction and the pressure on the surface of the cylinder are given in Table 1. One can see that for values of s $\geqslant$ 0.2 the results for the different approximations disagree; therefore, only $u^{(1)}(\eta)$ is presented here (dashed-dot curve in Fig. 1); it is obvious that as the rotational velocity increases, the longitudinal velocity profile is filled out. This circumstance is explained by the fact that a negative component of the pressure gradient in the direction Ox, which is determined by the function $p^{(1)}(\eta)$, appears in the boundary layer (Fig. 2). A series in powers of ξ is used in [2] to analyze the effect of transverse curvature in the case of a nonrotating cylinder. The numerical results obtained with its help are also applicable only for small values of ξ ($\xi \leqslant 0.5$).

Replacement of the variables

$$t = (\nu x/\omega R^3)^{2/5}; \quad \zeta = (r^2 - R^2)/2tR^2; \qquad (6)$$
$$\psi = \omega R^3 t^{3/2} f; \quad w^* = \omega R W; \quad p^* = p_\infty + \rho\omega^2 R^2 t P$$

in the case of large circumferential Reynolds numbers $Re^\omega = \omega R^2/\nu$ reduces the defining system of equations to the form

$$5(\sigma f_{\zeta\zeta})_\zeta + 3ff_{\zeta\zeta} - f_\zeta^2 + 2(\zeta P_\zeta - P) = 2t(f_\zeta f_{\zeta t} - f_t f_{\zeta\zeta} + P_t);$$
$$5(\sigma W_\zeta)_\zeta + 3fW_\zeta + (t/\sigma)(3f + 2tf_t - 2\zeta f_\zeta - 5t)W = 2t(f_\zeta W_t - f_t W_\zeta),$$
$$P_\zeta = W^2/\sigma, \text{where } \sigma = 1 + 2t\zeta.$$

Such a replacement is applicable to the solution of the problem of a rotating semiinfinite cylinder in a stationary liquid, since the velocity of the advancing flow is not used as a parameter in it; satisfaction of the boundary conditions

$$f_\zeta = f = 0, \; W = 1 \text{ when } \zeta = 0; \qquad (7)$$
$$f_\zeta = W = P = 0 \quad \text{as} \quad \zeta \to \infty$$

is required. One can seek the solution in the form of a series in powers of t when t is small. The principal part of the expansion is determined by the equations

$$5f_{\zeta\zeta\zeta} + 3ff_{\zeta\zeta} - f_\zeta^2 + 2(\zeta P_\zeta - P = 0; \qquad (8)$$
$$5W_{\zeta\zeta} + 3fW_\zeta = 0; \; P_\zeta = W^2.$$

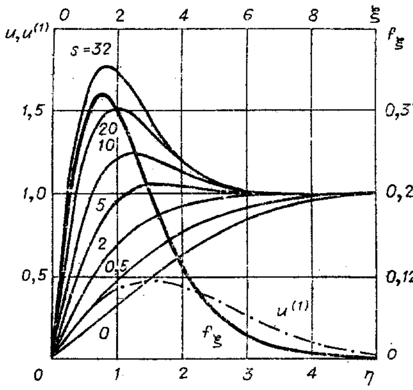

Fig. 1

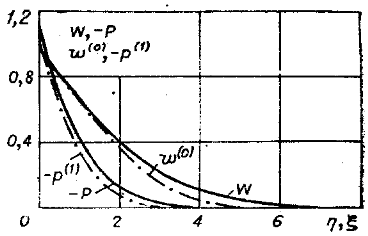

Fig. 2

The system (8) together with the boundary conditions (7) was solved numerically by the quasilinearization method [4]. The linear boundary-value problems were reduced, just as in the preceding case of small s, to the Cauchy problem for the fundamental solutions, which were integrated by the Runge–Kutta method. The results are presented in Figs. 1 and 2 and are supplemented by the constants $f_{\zeta\zeta 0} = 0.5834$, $W_{\zeta 0} = -0.3178$, and $P_0 = -1.1505$; the index 0 specifies the value on the surface of the cylinder. The problem of a semiinfinite cylinder rotating in a stationary liquid has not been completely solved in this paper. However, the results obtained show that when $Re^{\omega} \gg 1$, a boundary layer, whose thickness increases as $x^{2/5}$, is formed in the region of the leading edge of the cylinder, and the profile of the circumferential velocity differs insignificantly in shape from the Blasius profile (see Fig. 2), and mainly the longitudinal velocity component (thick curve in Fig. 1), whose maximum value increases as $x^{1/5}$, occurs in it.

Returning to the problem posed initially, one can show that the solution obtained to the problem (8) and (7) is applicable here in the case of large rotational velocities ($s \to \infty$, $\xi = \text{const}$). The connection of the variables (6) to the variables (1) adopted earlier is realized by the relations

$$\zeta = s^{1/5}\eta;\; \varphi = s^{1/5}f;\; p = s^{4/5}P;\; w = W.$$

The failure to satisfy the condition $\varphi_\eta = 1$ as $\eta \to \infty$ is not significant, since the flow is determined mainly by the part of the boundary layer next to the wall. It is evident that the maximum value of the longitudinal velocity component in any cross section of the boundary layer specified in advance begins to exceed the velocity of the advancing flow in the case of a sufficiently large ω and increases indefinitely as $\omega^{4/5}$ upon a further increase in the rotational velocity. The pressure coefficient increases in absolute value as $\omega^{8/5}$, i.e., faster yet.

The system (3) has a particular solution which satisfies the boundary conditions (4) but not the condition of the uniformity of the advancing flow,

$$w = 1/\sqrt{1+2\xi\eta} = R/r;\; p = -\beta/[2(1+2\xi\eta)] = -(\beta/2)(R/r)^2; \tag{9}$$

$$\varphi = \varphi^0, \tag{10}$$

where φ^0 is the solution of the problem of the flow around a nonrotating semiinfinite cylinder, and the relations (9) define the flow around a semiinfinite rotating cylinder. This particular solution is asymptotic for $\xi \to \infty$ and $\beta = \text{const}$ for the problem posed here. Thus the flow at a rather large distance from the leading edge can be assumed to be the superposition of two independent flows: a circumferential one (9) and a longitudinal one (10).

The asymptotic behavior of φ^0 as $\xi \to \infty$ was first investigated in [5]. The flow stress at the surface determined by the solution [5] falls off as $(\ln x)^{-1}$, and the velocity profiles are monotonic. The latter circumstance in the problem with rotation is explained by the absence of a longitudinal pressure gradient and permits drawing the following conclusion. If for a specified β a region occurs in the boundary layer in which the longitudinal velocity component exceeds the velocity of the advancing flow, this region is restricted in x and the longitudinal velocity in it attains its own maximum value.

A two-level difference scheme with implicit approximation of derivatives with respect to s ($a_s = [a(s) - a(s-\Delta s)]/\Delta s$) and quasilinearization was set up for the numerical solution of the problem (2) and (4). The method of solution for the system of ordinary linear differential equations represents an extension of the method described in [6] to the case of a system of higher order (in this case, the sixth).

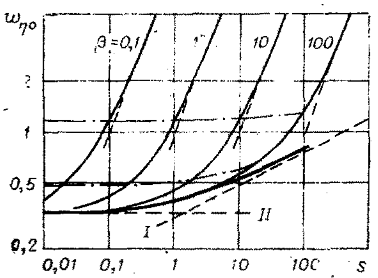

Fig. 3

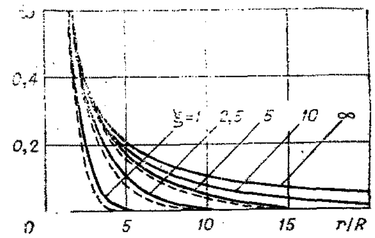

Fig. 4

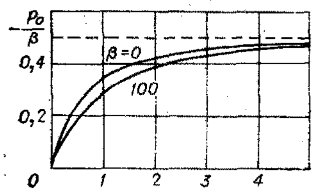

Fig. 5

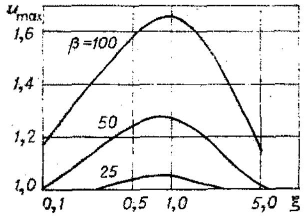

Fig. 6

The distributions of the longitudinal velocity component are presented in Fig. 1 for the case of a thin boundary layer (5) in the 0–32 range of variation of s. It is obvious that as the rotational velocity increases the profiles are continually filled out, and for s > 4 the longitudinal velocity in some region inside the boundary layer exceeds the velocity of the advancing flow. The scale for ζ and f_ζ is chosen so that one can compare the asymptotic velocity profile with the velocity profile for s = 32. Although quantitative agreement is observed only in the part near the wall (as should be expected) and the values of the maxima differ by more than 10%, the general trend and the decrease in the disagreement with a further increase in s (the calculation was carried out to s = 100) indicate that the asymptotic solution at infinitely large rotational velocities was found correctly. This conclusion confirms the results for the curcumferential velocity and the pressure. The shapes of the profiles of these quantities vary insignificantly with a change in s and are intermediate betweeen the limiting profiles for s = 0 and $s \to \infty$ shown in Fig. 2. The thick curve in Fig. 3, which illustrates the behavior of the derivative of the circumferential velocity on the surface of the cylinder at $\xi = 0$, deviates from the straight line II, which is determined by the constant value $-w_{\eta 0} = 0.332$ at s = 0, and asymptotically approaches the relation $-w_{\eta 0} = 0.318 \cdot s^{1/5}$, which is illustrated by the straight line I in the logarithmic grid.

The dashed-dot curves in Fig. 3 define the dependence of $w_{\eta 0}$ on the rotational velocity in a rather distant cross section of the boundary layer in which one cannot assume the thickness of the boundary layer to be infinitely small; $\xi = 1$ corresponds to the upper curve, and $\xi = 0.2$ to the lower curve. It is obvious that the convergence of the solutions of the system (3) to the solution of the system (5) as $\xi \to 0$ is uniform (the convergence is not uniform as $\beta \to \infty$).

The solid curves, which define the dependence of $w_{\eta 0}$ on x for various rotational velocities, begin to deviate from the limiting curve, then approach it, and at $\xi \approx 5$ they practically merge with the dashed straight lines $-w_{\eta 0} = s/\beta$, indicating that the relations (9) are actually asymptotic for $\xi \to \infty$ and β = const. Figure 4, in which the convergence of the profiles of the circumferential velocity to the hyperbola w = R/r far from the leading edge of the cylinder is shown, gives more direct confirmation. The dashed lines correspond to $\beta = 100$ and the solid curves, to $\beta \leq 10$; convergence is observed for any β.

The pressure distribution along the generating line of the cylinder, which is presented in Fig. 5, shows that it decreases monotonically; nowhere, however, does it become less than its own asymptotic value $p^* = p_\infty - \rho\omega^2 R^2/2$, which corresponds to the pressure on the surface of a rotating infinite cylinder.

The behavior of the profiles of the longitudinal velocity component in the case of an increase in s and finite values of ξ is analogous in general to their behavior in the case of a thin boundary layer. The difference lies in the slower filling out of the profiles at relatively low rotational velocities. The results for $\beta = 0.1$ differ by no more than 1% from the results of the numerical solution of the problem for a nonrotating cylinder [7] and are in complete agreement with the latter results for $\beta = 0.01$. In the case of $\beta \geq 20$ there exists a region in which the longitudinal velocity component exceeds the velocity of the advancing flow; the boundedness of this region with respect to x is confirmed by Fig. 6, in which it is also shown that u attains its own maximum value in the boundary layer at $\xi \approx 1$. In the case $\beta \leq 15$ the profiles of u are monotonic; the degree of influence of rotation on the longitudinal velocity field can be determined by the maximum value of the difference in the values of the longitudinal velocities for a rotating and a nonrotating cylinder at each specified cross section of the boundary layer; it turns out that this quantity first increases (in proportion to its distance from the leading edge of the cylinder), then decreases, acquiring its maximum value at $\xi \leqslant 1$.

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MOTION OF A SPHERICAL SOLID PARTICLE IN A NONUNIFORM FLOW OF A VISCOUS INCOMPRESSIBLE LIQUID

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UDC 532.582.7+532.516.5

The effect of a particle on the basic flow is studied, and the equations of motion of the particle are formulated. The problem is solved in the Stokes approximation with an accuracy up to the cube of the ratio of the radius of the sphere to the distance from the center of the sphere to peculiarities in the basic flow. An analogous problem concerning the motion of a sphere in a nonuniform flow of an ideal liquid has been discussed in [1]. We note that the solution is known in the case of flow around two spheres by a uniform flow of a viscous incompressible liquid [2], and we also note the papers [3, 4] on the motion of a small particle in a cylindrical tube.

Let us consider the slow flow (without a particle) of a viscous incompressible liquid. Let y_i be a fixed coordinate system; then the velocity and pressure of this flow will satisfy the equations

$$\mu \sum_{j=1}^{3} \frac{\partial^2 u_i^0}{\partial y_j^2} = \frac{\partial p^0}{\partial y_i}; \quad \sum_{i=1}^{3} \frac{\partial u_i^0}{\partial y_i} = 0, \tag{1}$$

where u_i^0 are the projections of the velocity vector onto the coordinate axes y_i; p^0 is the hydrodynamic pressure; u is the dynamic modulus of viscosity; and i = 1, 2, 3.

Let us introduce a new coordinate system x_i, whose center has the coordinates q_i in the coordinate system y_i. The relation between the coordinates is of the form

$$y_i = x_i + q_i.$$

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